hep-th/dddmmyy
February 2006

Why Randall-Sundrum Scenarious are not Compatible with an Orbifolded Fifth Dimension

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Abstract

We show the (already known) fact that Randall-Sundrum scenarious although compatible with a $\mathbb{Z}_2$ orbifold symmetry cannot hold regularity of the fields at the orbifold planes in the absence of boundary actions and respective jumps of the fields. This makes the models mathematical inconsistence and invalidate the inclusion of such models in a higher dimensional theory such as string theory. For completeness we point out some directions already in the literature and in progress.

Dedicated to a friend: Ian I. Kogan (not to be published)

The brane world concept was first introduced by Akama [1] in 1982 (see also [2–4]). Later such scenarious were also considered in [7] and exploit in a very simplistic manner by Randall and Sundrum [8, 9] using warpped geometries $M = M_4 \times (S_1/\mathbb{Z}_2)$ [RSI] and $M = M_4 \times (\mathbb{R}/\mathbb{Z}_2)$ [RSII]. The geometry used on these approachs is

$$ds^2 = K(y)(-dt^2 + dx^i dx_i) + dy^2 \ .$$

The solution of the eom compatible with the orbifold symmetry $\mathbb{Z}_2$ is

$$K(y) = \exp\{-k|y - y_0|\} \ .$$

For RSI, in order for this solution to be compatible with a compact coordinate it is necessary to sew together segments as shown in figure 1. The main problem is that it is not possible to have a regular field at all the

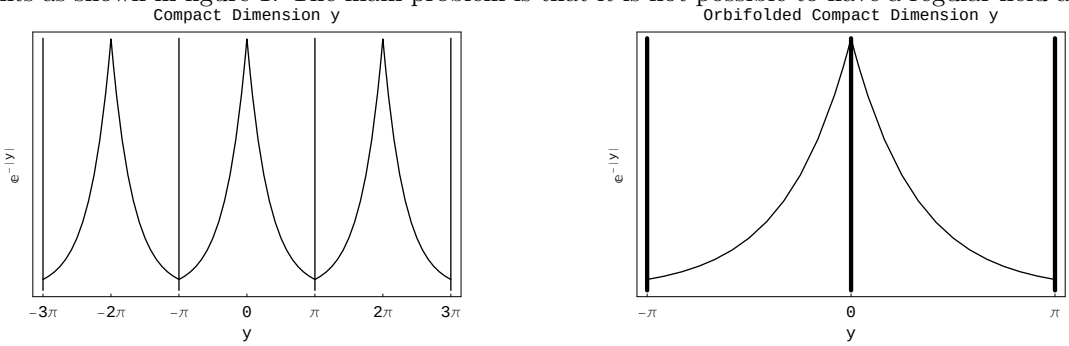

Figure 1: RSI solutions on a compact dimension (the thin vertical lines represent the interval identifications) and respective orbifold (the thick lines represent the orbifold planes.)

points of a compact coordinate. Upon orbifolding under $\mathbb{Z}_2$ we still have the same problem. At model level the usual approach is to consider appropriate brane actions localized at the orbifold planes that justify the jumps on the derivatives.

As for RSII the only solution compatible with $\mathbb{Z}_2$ symmetry is $K(y) = \exp\{-k|y|\}$. Again this field is not regular at $y = 0$. Again at model level upon orbifolding and considering appropriate boundary actions we can justify the jumps on the derivatives.

So the main drawback of RS scenarious is that, at most, can be considered at model level and cannot be included in any more fundamental higher theory as M/string-theory.

In the context of brane worlds there are already several works in the literature that give solutions for the problem presented here:

- Not considering an orbifold symmetry at all [10–12].
- By considering aditional scalar fields and perturbing the RS solution for $K(y)$ obtaining a regular solution at the orbifold points [13, 14].
- Considering FRW type of geometries and enlarged gauge groups [15, 16] that allow for periodic solutions.

This last approach further deserves at least to consider generic $N(y) \neq 1$ in order to understand if there are significative changes in the results obtained. We stress again that a warpped geometry *a la* RS is not an option.

Work supported by SFRH/BPD/17683/2004.

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